RESEARCH NOTE

Combined forced and free convective flow of water at 4 °C past a semi-infinite vertical plate

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Effects of buoyancy forces on forced and free convective flow of water at 4 °C past a semi-infinite vertical plate at constant temperature are studied. Flow is assumed to be vertically upwards. Similarity solutions are derived and the resulting equations are solved numerically on a computer. Velocity and temperature profiles are shown graphically and numerical values of the skin friction and the rate of heat transfer are entered in tables. It is observed that the skin friction and the Nusselt number increase with increasing Gr/Re^2, where Gr is the Grashof number and Re is the Reynolds number

Keywords: *convection, flow, water*

The study of combined forced and free convective flow of fluids has attracted a number of researchers, including Sparrow *et al*[1,2], Szewczyk[3], Merkin[4] and Acrivos[5]. Sparrow *et al* have presented a similarity solution for the combined effect of forced and free convection[1] and have given[2] the solution in power series in Gr/Re^2. Buoyancy effects in forced convective laminar flow have been analysed by a series method[3,4] and by an integral method[5]. In all these studies, the fluids (air and water) considered were at a normal temperature of 20 °C. Under these conditions, the variation in density is linear with respect to temperature, ie $\Delta\rho = \rho\beta(T_W - T_\infty)$. But when the water is at 4 °C, its density is maximum and hence the variation of ρ can only adequately be represented[6] by $\Delta\rho = \rho\gamma(T_W - T_\infty)^2$, where $\gamma = 8.0\times10^{-6}\,(°C)^{-2}$. Goren[6] studied the free convective flow of water at 4 °C past a semi-infinite plate by the similarity method. Soundalgekar[7] showed that a similarity solution exists for free convective flow of water at 4 °C. Gebhart and Mollendorf[8,9] have also studied the free convective flow of water at 4 °C under different conditions of $\Delta\rho$. However, combined free and forced convective flow of water at 4 °C appears as yet not to have been studied. In this study forced and free convective flow of water at 4 °C past a semi-infinite vertical plate is considered. Similarity solutions were derived for small values of Gr/Re^2, where the Grashof number $Gr = g\gamma(T_W - T_\infty)^2 x^3/\nu^2$ and the Reynolds number $Re = U_0 x/\nu$. The resulting equations were solved numerically on a computer and velocity and temperature profiles obtained. These results require comparison with experimental results when they become available.

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Mathematical analysis

Consider a two-dimensional flow of water at 4 °C past a semi-infinite vertical plate. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to it with the origin at the leading edge of the plate. Then, under the usual Boussinesq approximation, flow of water at 4 °C can be shown to be governed by the following equations:

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g(T - T_\infty)^2 \tag{2}$$

Energy:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

The boundary conditions are:

$$\begin{aligned} &u = 0,\ v = 0,\ T = T_W\ (\gtrless 4\,°C)\ \text{at}\ y = 0 \\ &u = U_\infty,\ v \to 0,\ T \to T_\infty\ (= 4\,°C)\ \text{as}\ y \to \infty \end{aligned} \tag{4}$$

Here u, v are the velocity components along x and y directions, ν is the kinematic viscosity, g the acceleration due to gravity, T the temperature in the boundary layer, $\alpha = k/\rho c_p$ (the thermal diffusivity), T_W the plate temperature, T_∞ the free stream temperature of the water, U_∞ the free-stream velocity, k the thermal conductivity and c_p the specific heat at constant pressure.

We now introduce the following transformations:

$$\eta = y\left(\frac{U_\infty}{\nu x}\right)^{1/2}, \quad u = U_\infty f', \quad v = \frac{(U_\infty \nu x)^{1/2}}{2x}(\eta f' - f), \tag{5}$$

$$\theta = \frac{T - T_\infty}{T_W - T_\infty}, \quad Pr = \frac{\mu c_p}{k}$$

into Eqs (2) and (3) and obtain

$$f''' + \tfrac{1}{2}ff'' = -\frac{Gr}{Re^2}\theta^2 \tag{6}$$

$$\theta'' + \tfrac{1}{2}Prf\theta' = 0 \tag{7}$$

with the boundary conditions

$$\begin{aligned} &f(0) = 0, \quad f'(0) = 0, \quad \theta = 1 \quad \text{at } \eta = 0 \\ &f'(\infty) = 1, \quad \theta(\infty) = 0 \quad \text{as } \eta \to \infty \end{aligned} \tag{8}$$

Here, Pr is the Prandtl number and Gr/Re^2 is defined as $\{g\gamma(T_W - T_\infty)^2 x^3/\nu^2\}/\{U_\infty^2 x^2/\nu^2\}$ where Gr is the Grashof number and Re the Reynolds number. Because Gr is a function of x, Eq (6) is not a similarity equation. But for the present problem, Gr/Re^2 is always small (<1), hence f and θ can be expanded in powers of Gr/Re^2 as

$$\left.\begin{aligned} f &= f_0 + \frac{Gr}{Re^2}f_1 + 0(Gr/Re^2) \\ \theta &= \theta_0 + \frac{Gr}{Re^2}\theta_1 + 0(Gr/Re^2) \end{aligned}\right\} \tag{9}$$

Substituting Eq. (9) in Eqs (6) and (7), equating the coefficients of different powers of Gr/Re^2 and neglecting those of order $(Gr/Re^2)^2$ and higher, the following equations are obtained:

$$f_0''' + \tfrac{1}{2}f_0 + f_0'' = 0 \tag{10}$$

$$f_1''' + \tfrac{1}{2}(f_0 f_1'' + f_1 f_0'') = -\theta_0^2 \tag{11}$$

$$\theta_0'' + \tfrac{1}{2}\, Pr\, f_0\theta_0' = 0 \tag{12}$$

$$\theta_1'' + \tfrac{1}{2}\, Pr\,(f_0\theta_1' + f_1\theta_0') = 0. \tag{13}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} &f_0(0) = 0, \quad f_1(0) = 0, \quad f_0'(0) = 0, \quad f_1'(0) = 0 \\ &\theta_0(0) = 1, \quad \theta_1(0) = 0 \\ &f_0'(\infty) = 1, \quad f_1'(\infty) = 0, \quad \theta_0(\infty) = 0, \quad \theta_1(\infty) = 0 \end{aligned}\right\} \tag{14}$$

Eqs (10)–(13), subject to the boundary conditions (14), constitute a set of non-linear similarity-type equations and these were solved numerically on a computer. The velocity profiles are shown in Fig. 1 for different values of Gr/Re^2 and $Pr = 11.4$, which is the Prandtl number for water at 4 °C. It can be seen from this figure that the velocity increases with increasing values of Gr/Re^2. In Fig. 2, the temperature profiles are shown. It can be seen that an increase in Gr/Re^2 leads to a decrease in the temperature of water at 4 °C.

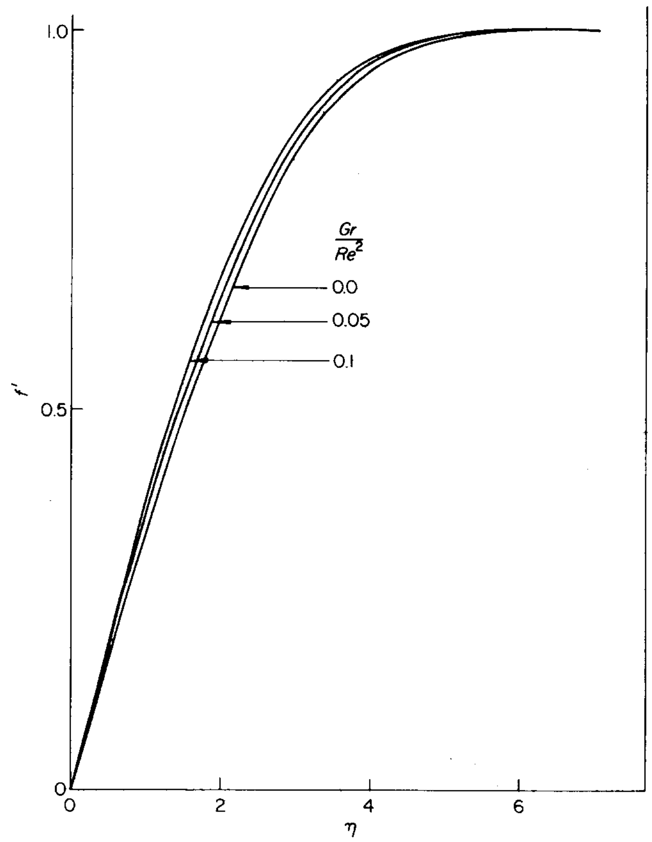

Fig 1

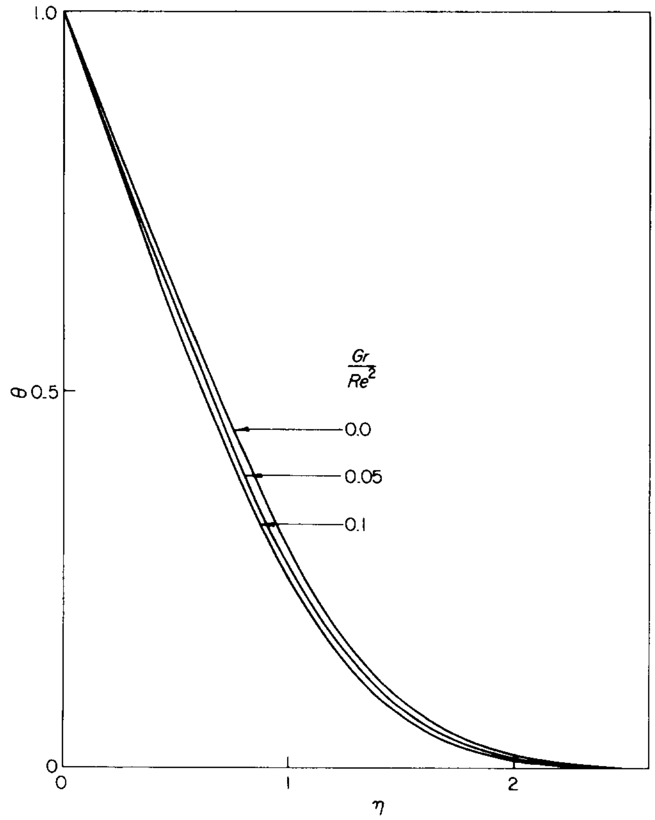

Fig 2

Table 1 Values of $f''(\eta)$ for $Pr = 11.4$

η	Gr/Re^2 0.0	0.05	0.1	0.2
0.0	0.3320	0.3543	0.3766	0.4211
0.2	0.3319	0.3457	0.3594	0.3868
0.4	0.3314	0.3391	0.3468	0.3622
0.6	0.3300	0.3337	0.3374	0.3448
0.8	0.3274	0.3286	0.3298	0.3323
1.0	0.3230	0.3228	0.3226	0.3222
1.2	0.3165	0.3156	0.3146	0.3127
1.4	0.3078	0.3064	0.3050	0.3022
1.6	0.2966	0.2949	0.2932	0.2898
1.8	0.2829	0.2809	0.2790	0.2751
2.0	0.2667	0.2646	0.2624	0.2581
3.0	0.1613	0.1588	0.1563	0.1514
4.0	0.0642	0.0626	0.0610	0.0592
5.0	0.0159	0.0153	0.0148	0.0137
6.0	0.0024	0.0022	0.0021	0.0019
7.0	0.0002	0.0002	0.0001	0.0001

Table 2 Values of $\{-\theta'(\eta)\}$ for $Pr = 11.4$

η	Gr/Re^2 0.0	0.05	0.1	0.2
0.0	0.7608	0.7691	0.7773	0.7938
0.2	0.7589	0.7670	0.7751	0.7913
0.4	0.7456	0.7529	0.7602	0.7747
0.6	0.7107	0.7161	0.7214	0.7322
0.8	0.6474	0.6499	0.6523	0.6573
1.0	0.5552	0.5543	0.5535	0.5517
1.2	0.4416	0.4378	0.4340	0.4264
1.4	0.3212	0.2156	0.3100	0.2989
1.6	0.2105	0.2047	0.1989	0.1872
1.8	0.1226	0.1178	0.1129	0.1033
2.0	0.0626	0.0593	0.0560	0.0494
2.2	0.0277	0.0258	0.0240	0.0202
2.4	0.0105	0.0096	0.0087	0.0702
2.6	0.0033	0.0030	0.0027	0.0020
2.8	0.0009	0.0008	0.0006	0.0004
3.0	0.0002	0.0001	0.0001	0.0000

With a knowledge of the velocity field, skin friction can be derived in non-dimensional form as

$$\tau = (2/(Re)^{1/2})f''(0) \quad (15)$$

where $\tau = \tau'/(\frac{1}{2}\rho U_\infty^2)$. The numerical values of $f''(0)$ are given in Table 1, from which it can be concluded that an increase in the value of Gr/Re^2 leads to an increase in the value of the skin friction.

The rate of heat transfer is given by

$$q(x) = -k\frac{\partial T}{\partial y}\bigg|_{y=0} \quad (16)$$

The heat transfer coefficient is defined by

$$h = \frac{q(x)}{T_W - T_\infty} = -k[U_\infty/(\nu x)]^{1/2}\theta'(0)$$

and the Nusselt number by

$$Nu = \frac{hx}{k} = -(Re)^{1/2}\theta'(0) \quad (17)$$

The numerical values of $\{-\theta'(0)\}$ were calculated and are given in Table 2. It is seen that an increase in the value of Gr/Re^2 leads to an increase in the value of the Nusselt number.

Conclusions

An increase in the value of Gr/Re^2 leads to an increase in the velocity and a decrease in the temperature of water.

An increase in the value of Gr/Re^2 leads to an increase in both the skin friction and the rate of heat transfer.

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References

1. **Sparrow E. M., Eichhorn R. and Gregg J. L.** Combined forced and free convection in a boundary layer flow. *Physics of Fluids*, 1959, **2**, 319
2. **Sparrow E. M. and Gregg J. L.** Buoyancy effects in forced convection flow and heat transfer. *J. Applied Mech.* (*Tr. ASME*), 1959, **81E**, 133
3. **Szewczyk A. A.** Combined forced and free convection laminar flow. *J. Heat Transfer* (*Tr. ASME*), 1964, **86C**, 501
4. **Merkin J. H.** The effect of buoyancy forces on the boundary layer flow over a semi-infinite flat plate in a uniform free-stream, *J. Fluid Mech.*, 1969, **35**, 439
5. **Acrivos A.** Combined laminar free and forced convection heat transfer in external flows. *J. AIChE*, 1958, **4**, 285
6. **Goren S. L.** On free convection in water at 4 °C. *Chem. Eng. Sci.*, 1966, **21**, 515
7. **Soundalgekar V. M.** Laminar free convection flow of water at 4 °C from a vertical plate with variable wall temperature. *Chem. Eng. Sci.*, 1973, **28**, 307
8. **Gebhart B. and Mollendorf J. C.** A new density relation for pure and saline water. *Deep Sea Res.*, 1977, **24**, 831
9. **Gebhart B. and Mollendorf J. C.** Buoyancy induced flows in water under conditions in which density extrema may arise. *J. Fluid Mech.*, 1978, **89**, 673